Homework #3 (100 points) - Show all work on the following problems:
(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (15 points): Consider an electric field $\vec{E} = kr^3\hat{r}$ (with k a constant):

a. Find the charge density ρ(r) as function of position, using Gauss's law (differential form).

b. Find the total charge Q contained in a sphere of radius R, by using Gauss's law (integral form).

c. Find the total charge Q contained in a sphere of radius R, by direct integration of the charge density.

Problem 2 (15 points): Find the electric field as a function of radius r inside a sphere with uniform charge density ρ throughout.

Problem 3 (15 points): Find the electric field as a function of radius r inside a sphere with charge density that increases linearly from the origin (ρ(r) = kr).

Problem 4 (30 points): One of the following vector functions is not a valid electrostatic field. By evaluating the curl, determine which one is impossible. For the one that is a valid electrostatic field, find the corresponding electric potential V.

a. $\vec{E} = xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}$
b. $\vec{E} = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$

Problem 5 (25 points): Find the electric potential V(r) as a function of radius inside and outside of a sphere with uniform charge density throughout, with radius R and total charge Q, by integrating from infinity. Explicitly compute the gradient of this function and double-check that $-\nabla V$ gives the correct electric field inside and outside the sphere.